CORK INSTITUTE OF TECHNOLOGY
INSTITIÚID TEICNEOLAÍOCHTA CHORCAÍ

MATHEMATICS

The Lee Fields Medal
A CIT Mathematics Competition

1 Introduction

Save for a Higher Diploma and a Masters in Data Science & Analytics, the Department of Mathematics at Cork Institute of Technology is primarily in the business of service teaching, with the mathematics taught taking an applied and sometimes even vocational slant. However, conceived by Michael Brennan, for a number of years the Department has offered a module — MATH6028 Mathematical Explorations — that students can take as a free choice elective. As can be seen from the module descriptor:

> *The objective of this module is to capture the beauty and power of mathematics through various explorations,*

this module provides the CIT student with something very different to their programme-aligned mathematics modules. Mathematical Explorations runs in both Semesters I and II, and is always enrolled to capacity. The ongoing popularity of Mathematical Explorations proves that there is an appetite amongst CIT students for more mathematics, mathematics for enjoyment.

The other side of the coin is that there can sometimes be a feeling within our department, and perhaps further afield in the IoT sector in general, that while we (rightfully) expend a lot of our energy on student retention, and on those struggling, that perhaps some of our students that are more interested and more capable in mathematics might be missing out on some attention.

For these reasons, the CIT Department of Mathematics established a mathematics competition, open to all currently registered CIT students. Called the Lee Fields Medal, the contest consists of a paper of ten questions, based on mathematics no more advanced than Ordinary Level Leaving Certificate Mathematics.

2 A Call-to-Arms

Are you lecturing mathematics in an Irish IoT? Do you feel some of your students would appreciate a similar outlet? Perhaps, and you don't necessarily need a local competition to do this, you would be interested in setting up an inter-IoT mathematics competition? If yes, please get in touch with J.P. McCarthy, `jeremiah.mccarthy@cit.ie`

3 Years One & Two

The inaugural competition was held in October 2018, where 18 intrepid students sat the paper. An evening Software Development student, Damien Murphy, prevailed with a fine score of 83%. The toughest question on the paper was the Birthday Problem. Paschal Mullins, a first year student of Mechanical Engineering (Hons) was the only student to get full marks in that question, and went on to win the best first year prize with a score of 74%.

In October 2019 the competition returned, and 25 students took the paper. The winner of Best First Year in 2018, Paschal Mullins, returned and with 83% took down the title. On this occasion the toughest and second toughest questions were the geometry questions, and Shane Allen, a third year Level 7 Mechanical Engineering student was the only entrant to get full marks on both questions. To encourage more Level 7 students to enter, we added a Best Level 7 prize, and Shane won this award to go with the honour of cracking the two geometry problems. A first year computer science student, Yi Ming Tan, came second overall with a mark of 79%, and so the Best First Year student went to the next highest ranked first year, Sofia Dolera Perez, an Electronic Engineering (Hons) student with a score of 70%.

See `https://mathematics.cit.ie/let_s-do-maths` for the 2018 and 2019 papers.

4 Organisation

The competition is organised in a collaborative manner by an organising committee (OC). A bank of questions in ten categories has been developed, and there is an annual call to departmental colleagues to submit further questions. The competition is held relatively early in Semester I, with students invited via an "all-students" email. The paper is chosen democratically, with two rounds of voting. First the OC each pick three questions from each category, and then the two top-voted questions are ran off against each other in a second vote. On the night the students receive a pack containing instructional cover sheet, question paper, ten answer sheets, and formula booklet. We also use the opportunity to advertise elective modules run by the department with a brochure.

Key to the process are the answer sheets: only one question per sheet, and this makes the process of divvying up the corrections very straightforward. Members of the OC each mark two or three questions, and submit marks to an online spreadsheet. Two weeks after the students sit the paper, we have a Results & Solutions night where the prizes are presented.

Furthermore, on the night of CIT Faculty of Engineering and Science Awards, the winner is presented with a rather fetching gold medal, suitably inscribed with Euler's Identity.

Figure 1: Paschal Mullins, 2nd Year Mechanical Engineering (Hons), receiving the Lee Fields Medal at the CIT Faculty of Science and Engineering Awards from J.P. McCarthy (left) and HoD David Goulding (right).

Dr Michael Brennan: michael.brennan@cit.ie
Dr David Goulding: david.goulding@cit.ie
Dr Seán Lacey: sean.lacey@cit.ie
Dr J.P. McCarthy: jeremiah.mccarthy@cit.ie
Dr Violeta Morari: violeta.morari@cit.ie
Dr Catherine Palmer: catherine.palmer@cit.ie

Lee Fields Organising Committee, Department of Mathematics, CIT.

The Lee Fields Medal

Inaugural Competition, Maths Week 2018

Department of Mathematics, CIT

Time Allowed: Up to Three Hours

Tables and calculators may be used.

Answer <u>all</u> ten questions

1. Using each number exactly once, place the numbers 1, 2, 3, 4, 5, 6, 7, 8, and 9 in a $3{\times}3$ square so that the rows, columns and diagonals sum to the same total.

2. Professor Oldie does not believe in calculators. You have to prove it to him on paper, using mathematical considerations, that

$$\sqrt{10} > \sqrt{2} + \sqrt{3}.$$

   You may not use approximations nor your calculator.

3. CIT students Rebecca and Aoife share an apartment which is 6 km away from college. One day they left the apartment at the same time but decided to walk to the college separately. Rebecca walked first half of the *distance* with the speed of 4 km/h and the second half of the *distance* at 2 km/h. Aoife had a different plan. She walked the first half of the *time* with the speed of 4 km/h and walked the remaining half of the *time* at 2 km/h. Who got to college first? Show all calculations.

4. Explain geometrically why the set of simultaneous equations

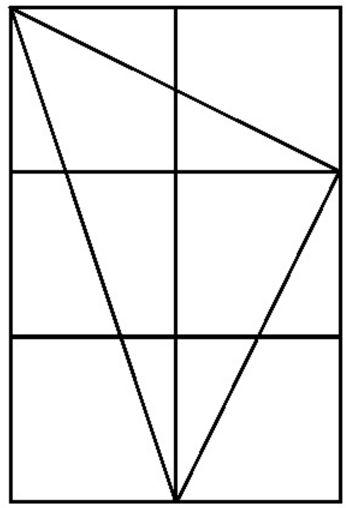
$$2x - y = -3$$
$$2x - y = -2$$

   has *no* solution.

5. Which is a closer fit, a square peg in a round hole, or a round peg in a square hole? Justify your answer fully.

6. Use the below figure to prove that

$$\tan^{-1}(1) + \tan^{-1}(2) + \tan^{-1}(3) = \pi = 180°.$$



7. If you have 23 people in a room, what is the probability that at least two of them share a birthday?

8. A piece of wire 10 m long is cut into two pieces. One piece is bent into a square and the other is bent into an equilateral triangle. How should the wire be cut so that the total area enclosed is maximised?

9. If every person in a group of 20 shook hands with all of the other people in the group, how many more handshakes take place than if the group were to split themselves into two groups of 10 and each person only shook hands with the other nine people in their group?

10. There are eight identical-looking coins; one of these coins is counterfeit and is known to be lighter than the genuine coins. What is the minimum number of weighings needed to identify the fake coin with a two-pan balance scale without weights?

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MATHEMATICS

The Lee Fields Medal II

TIME ALLOWED: UP TO THREE HOURS

TABLES AND CALCULATORS MAY BE USED.

ANSWER ALL TEN QUESTIONS

1. Steven is deep in thought looking for positive whole numbers m and n that satisfy the equation
$$20m + 19n = 2020.$$
That is easy says Julie, just take $m = 101$ and $n = 0$. Of course, she's correct. Can you find another solution where $0 < m < 10$?

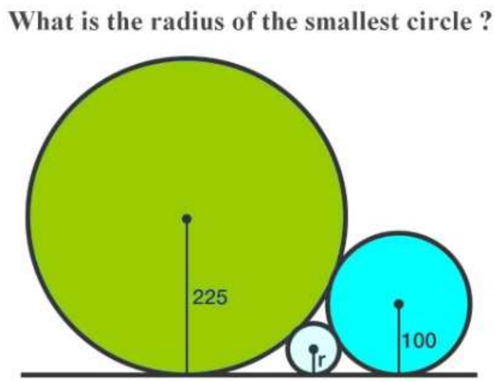
2. The following holds for any real numbers a, and r such that $|r| < 1$:
$$a + ar + ar^2 + ar^3 + \cdots = \frac{a}{1-r}$$
Hence, or otherwise, express
$$0.42424242\cdots$$
as a rational number.

3. Consider a row of n tiles, each tile of which is red or blue. Suppose that a blue tile is never followed by a blue tile, so that $RBRBRR$ is allowed, but $RBBRRB$ is not.

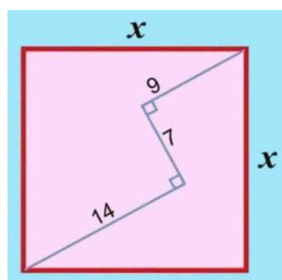
   Let $t(n)$ be the number of allowed tilings. Find $t(1)$, and $t(2)$, and come up with a formula for $t(n)$ in terms of $t(n-1)$ and $t(n-2)$.

4. Suppose that m and c are constants. What is the equation of the tangent to the graph $y = mx + c$ at $x = 1$?

5. Find r



6. Find x



7. In a game show you have to choose one of three doors. One conceals a new car and the other two contain angry lions who will attack you. You choose but your chosen door is not opened immediately. Instead the presenter tells you that another door (which you have not picked), contains a lion. You then have the opportunity to change your mind. Is there any benefit to doing so? Justify your answer.

8. Write down an expression for

$$\frac{d^{2019}}{dx^{2019}}x^{2019},$$

the 2019-th derivative of x^{2019}.

9. If you expand

$$(1+x)^{2019} = a_0 + a_1x + a_2x^2 + a_3x^3 + \cdots + a_{2019}x^{2019},$$

what is the coefficient of x^2, a_2?

10. Using a 5-litre container and a 7-litre container, what is the minimum supply of water you need to measure exactly 4 litres of water?